Integration - Average Value Of A Function

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Topic tags have been given for each question to enable you to know if you can do the question or whether you need to wait to cover the additional topic(s).

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Question 1

Qualification: AP Calculus AB

Areas: Applications of Differentiation, Differentiation, Applications of Integration, Integration

Subtopics: Mean Value Theorem, Interpreting Meaning in Applied Contexts, Rates of Change (Average), Riemann Sums – Trapezoidal Rule, Average Value of a Function, Differentiation Technique – Exponentials, Differentiation Technique – Product Rule

Paper: Part A-Calc / Series: 2001 / Difficulty: Hard / Question Number: 2

t (days)	$W(t)$ (°C)
0	20
3	31
6	28
9	24
12	22
15	21

2. The temperature, in degrees Celsius (°C), of the water in a pond is a differentiable function W of time t. The table above shows the water temperature as recorded every 3 days over a 15-day period.

(a) Use data from the table to find an approximation for $W'(12)$. Show the computations that lead to your answer. Indicate units of measure.

(b) Approximate the average temperature, in degrees Celsius, of the water over the time interval $0 \le t \le 15$ days by using a trapezoidal approximation with subintervals of length $\Delta t = 3$ days.

(c) A student proposes the function P, given by $P(t) = 20 + 10te^{(-t/3)}$, as a model for the temperature of the water in the pond at time t, where t is measured in days and $P(t)$ is measured in degrees Celsius. Find $P'(12)$. Using appropriate units, explain the meaning of your answer in terms of water temperature.

(d) Use the function P defined in part (c) to find the average value, in degrees Celsius, of $P(t)$ over the time interval $0 \le t \le 15$ days.

Question 2

Qualification: AP Calculus AB

Areas: Limits and Continuity, Integration, Applications of Integration

Subtopics: Continuities and Discontinuities, Calculating Limits Algebraically, Average Value of a Function, Properties of Integrals, Integration Technique – Standard Functions, Differentiability

Paper: Part B-Non-Calc / Series: 2003 / Difficulty: Very Hard / Question Number: 6

6. Let f be the function defined by

$$f(x) = \begin{cases} \sqrt{x+1} & \text{for } 0 \le x \le 3 \\ 5 - x & \text{for } 3 < x \le 5. \end{cases}$$

(a) Is f continuous at $x = 3$? Explain why or why not.

(b) Find the average value of $f(x)$ on the closed interval $0 \le x \le 5$.

(c) Suppose the function g is defined by

$$g(x) = \begin{cases} k\sqrt{x+1} & \text{for } 0 \le x \le 3 \\ mx + 2 & \text{for } 3 < x \le 5, \end{cases}$$

where k and m are constants. If g is differentiable at $x = 3$, what are the values of k and m?

Question 3

Qualification: AP Calculus AB

Areas: Applications of Integration, Integration, Applications of Differentiation

Subtopics: Average Value of a Function, Riemann Sums – Midpoint, Interpreting Meaning in Applied Contexts, Mean Value Theorem

Paper: Part A-Calc / Series: 2003-Form-B / Difficulty: Somewhat Challenging / Question Number: 3

Distance x (mm)	0	60	120	180	240	300	360
Diameter $B(x)$ (mm)	24	30	28	30	26	24	26

3. A blood vessel is 360 millimeters (mm) long with circular cross sections of varying diameter. The table above gives the measurements of the diameter of the blood vessel at selected points along the length of the blood vessel, where x represents the distance from one end of the blood vessel and $B(x)$ is a twice-differentiable function that represents the diameter at that point.

   (a) Write an integral expression in terms of $B(x)$ that represents the average radius, in mm, of the blood vessel between $x = 0$ and $x = 360$.

   (b) Approximate the value of your answer from part (a) using the data from the table and a midpoint Riemann sum with three subintervals of equal length. Show the computations that lead to your answer.

   (c) Using correct units, explain the meaning of $\pi\int_{125}^{275}\left(\frac{B(x)}{2}\right)^2 dx$ in terms of the blood vessel.

   (d) Explain why there must be at least one value x, for $0 < x < 360$, such that $B''(x) = 0$.

Question 4

Qualification: AP Calculus AB

Areas: Applications of Integration, Applications of Differentiation

Subtopics: Total Amount, Increasing/Decreasing , Average Value of a Function, Rates of Change (Average), Accumulation of Change

Paper: Part A-Calc / Series: 2004 / Difficulty: Easy / Question Number: 1

1. Traffic flow is defined as the rate at which cars pass through an intersection, measured in cars per minute. The traffic flow at a particular intersection is modeled by the function F defined by

$$F(t) = 82 + 4\sin\left(\frac{t}{2}\right) \text{ for } 0 \le t \le 30,$$

where $F(t)$ is measured in cars per minute and t is measured in minutes.

(a) To the nearest whole number, how many cars pass through the intersection over the 30-minute period?

(b) Is the traffic flow increasing or decreasing at $t = 7$? Give a reason for your answer.

(c) What is the average value of the traffic flow over the time interval $10 \le t \le 15$? Indicate units of measure.

(d) What is the average rate of change of the traffic flow over the time interval $10 \le t \le 15$? Indicate units of measure.

Question 5

Qualification: AP Calculus AB

Areas: Integration, Applications of Differentiation, Applications of Integration

Subtopics: Riemann Sums – Midpoint, Interpreting Meaning in Applied Contexts, Mean Value Theorem, Kinematics (Displacement, Velocity, and Acceleration), Average Value of a Function

Paper: Part A-Calc / Series: 2004-Form-B / Difficulty: Medium / Question Number: 3

t (minutes)	0	5	10	15	20	25	30	35	40
$v(t)$ (miles per minute)	7.0	9.2	9.5	7.0	4.5	2.4	2.4	4.3	7.3

3. A test plane flies in a straight line with positive velocity $v(t)$, in miles per minute at time t minutes, where v is a differentiable function of t. Selected values of $v(t)$ for $0 \le t \le 40$ are shown in the table above.

(a) Use a midpoint Riemann sum with four subintervals of equal length and values from the table to approximate $\int_0^{40} v(t)\,dt$. Show the computations that lead to your answer. Using correct units, explain the meaning of $\int_0^{40} v(t)\,dt$ in terms of the plane's flight.

(b) Based on the values in the table, what is the smallest number of instances at which the acceleration of the plane could equal zero on the open interval $0 < t < 40$? Justify your answer.

(c) The function f, defined by $f(t) = 6 + \cos\left(\frac{t}{10}\right) + 3\sin\left(\frac{7t}{40}\right)$, is used to model the velocity of the plane, in miles per minute, for $0 \le t \le 40$. According to this model, what is the acceleration of the plane at $t = 23$? Indicate units of measure.

(d) According to the model f, given in part (c), what is the average velocity of the plane, in miles per minute, over the time interval $0 \le t \le 40$?

Question 6

Qualification: AP Calculus AB

Areas: Differentiation, Integration, Applications of Integration, Applications of Differentiation

Subtopics: Rates of Change (Average), Riemann Sums – Trapezoidal Rule, Average Value of a Function, Fundamental Theorem of Calculus (First), Interpreting Meaning in Applied Contexts, Mean Value Theorem

Paper: Part A-Calc / Series: 2005 / Difficulty: Very Hard / Question Number: 3

Distance x (cm)	0	1	5	6	8
Temperature $T(x)$ (°C)	100	93	70	62	55

3. A metal wire of length 8 centimeters (cm) is heated at one end. The table above gives selected values of the temperature $T(x)$, in degrees Celsius (°C), of the wire x cm from the heated end. The function T is decreasing and twice differentiable.

(a) Estimate $T'(7)$. Show the work that leads to your answer. Indicate units of measure.

(b) Write an integral expression in terms of $T(x)$ for the average temperature of the wire. Estimate the average temperature of the wire using a trapezoidal sum with the four subintervals indicated by the data in the table. Indicate units of measure.

(c) Find $\int_0^8 T'(x)\,dx$, and indicate units of measure. Explain the meaning of $\int_0^8 T'(x)\,dx$ in terms of the temperature of the wire.

(d) Are the data in the table consistent with the assertion that $T''(x) > 0$ for every x in the interval $0 < x < 8$? Explain your answer.

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Question 7

Qualification: AP Calculus AB

Areas: Applications of Integration

Subtopics: Kinematics (Displacement, Velocity, and Acceleration), Average Value of a Function, Total Amount

Paper: Part A-Calc / Series: 2005-Form-B / Difficulty: Medium / Question Number: 3

3. A particle moves along the x-axis so that its velocity v at time t, for $0 \le t \le 5$, is given by $v(t) = \ln\left(t^2 - 3t + 3\right)$. The particle is at position $x = 8$ at time $t = 0$.

   (a) Find the acceleration of the particle at time $t = 4$.

   (b) Find all times t in the open interval $0 < t < 5$ at which the particle changes direction. During which time intervals, for $0 \le t \le 5$, does the particle travel to the left?

   (c) Find the position of the particle at time $t = 2$.

   (d) Find the average speed of the particle over the interval $0 \le t \le 2$.

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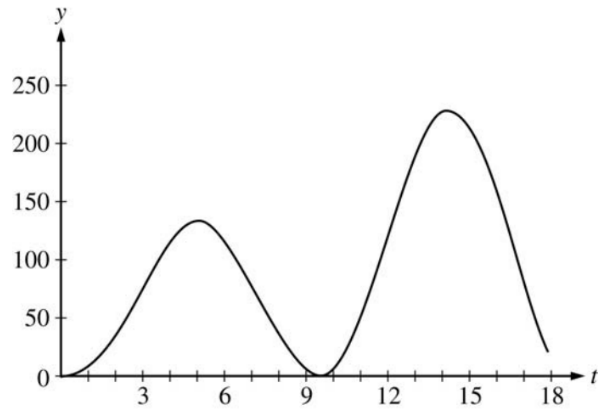
Question 8

Qualification: AP Calculus AB

Areas: Applications of Integration

Subtopics: Total Amount, Average Value of a Function, Interpreting Meaning in Applied Contexts

Paper: Part A-Calc / Series: 2006 / Difficulty: Somewhat Challenging / Question Number: 2



2. At an intersection in Thomasville, Oregon, cars turn left at the rate $L(t) = 60\sqrt{t}\sin^2\left(\frac{t}{3}\right)$ cars per hour over the time interval $0 \le t \le 18$ hours. The graph of $y = L(t)$ is shown above.

(a) To the nearest whole number, find the total number of cars turning left at the intersection over the time interval $0 \le t \le 18$ hours.

(b) Traffic engineers will consider turn restrictions when $L(t) \ge 150$ cars per hour. Find all values of t for which $L(t) \ge 150$ and compute the average value of L over this time interval. Indicate units of measure.

(c) Traffic engineers will install a signal if there is any two-hour time interval during which the product of the total number of cars turning left and the total number of oncoming cars traveling straight through the intersection is greater than 200,000. In every two-hour time interval, 500 oncoming cars travel straight through the intersection. Does this intersection require a traffic signal? Explain the reasoning that leads to your conclusion.

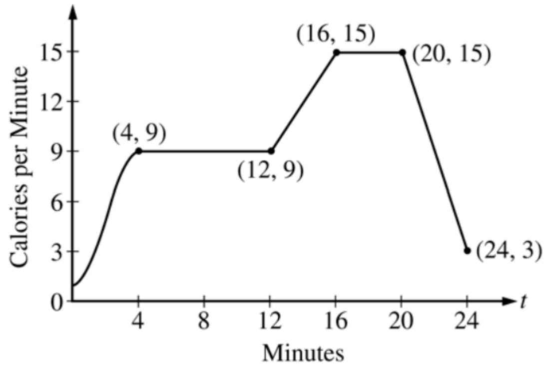
Question 9

Qualification: AP Calculus AB

Areas: Applications of Integration, Applications of Differentiation

Subtopics: Increasing/Decreasing , Rates of Change (Instantaneous), Global or Absolute Minima and Maxima, Total Amount, Accumulation of Change, Average Value of a Function

Paper: Part B-Non-Calc / Series: 2006-Form-B / Difficulty: Hard / Question Number: 4



4. The rate, in calories per minute, at which a person using an exercise machine burns calories is modeled by the function f. In the figure above, $f(t) = -\frac{1}{4}t^3 + \frac{3}{2}t^2 + 1$ for $0 \le t \le 4$ and f is piecewise linear for $4 \le t \le 24$.

(a) Find $f'(22)$. Indicate units of measure.

(b) For the time interval $0 \le t \le 24$, at what time t is f increasing at its greatest rate? Show the reasoning that supports your answer.

(c) Find the total number of calories burned over the time interval $6 \le t \le 18$ minutes.

(d) The setting on the machine is now changed so that the person burns $f(t) + c$ calories per minute. For this setting, find c so that an average of 15 calories per minute is burned during the time interval $6 \le t \le 18$.

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Question 10

Qualification: AP Calculus AB

Areas: Applications of Integration, Integration

Subtopics: Riemann Sums – Trapezoidal Rule, Average Value of a Function, Integration - Area Under A Curve, Interpreting Meaning in Applied Contexts, Modelling Situations

Paper: Part A-Calc / Series: 2008-Form-B / Difficulty: Medium / Question Number: 3

Distance from the river's edge (feet)	0	8	14	22	24
Depth of the water (feet)	0	7	8	2	0

3. A scientist measures the depth of the Doe River at Picnic Point. The river is 24 feet wide at this location. The measurements are taken in a straight line perpendicular to the edge of the river. The data are shown in the table above. The velocity of the water at Picnic Point, in feet per minute, is modeled by $v(t) = 16 + 2\sin\left(\sqrt{t+10}\right)$ for $0 \le t \le 120$ minutes.

(a) Use a trapezoidal sum with the four subintervals indicated by the data in the table to approximate the area of the cross section of the river at Picnic Point, in square feet. Show the computations that lead to your answer.

(b) The volumetric flow at a location along the river is the product of the cross-sectional area and the velocity of the water at that location. Use your approximation from part (a) to estimate the average value of the volumetric flow at Picnic Point, in cubic feet per minute, from $t = 0$ to $t = 120$ minutes.

(c) The scientist proposes the function f, given by $f(x) = 8\sin\left(\frac{\pi x}{24}\right)$, as a model for the depth of the water, in feet, at Picnic Point x feet from the river's edge. Find the area of the cross section of the river at Picnic Point based on this model.

(d) Recall that the volumetric flow is the product of the cross-sectional area and the velocity of the water at a location. To prevent flooding, water must be diverted if the average value of the volumetric flow at Picnic Point exceeds 2100 cubic feet per minute for a 20-minute period. Using your answer from part (c), find the average value of the volumetric flow during the time interval $40 \le t \le 60$ minutes. Does this value indicate that the water must be diverted?

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Question 11

Qualification: AP Calculus AB

Areas: Applications of Integration, Applications of Differentiation

Subtopics: Total Amount, Local or Relative Minima and Maxima, Fundamental Theorem of Calculus (First), Average Value of a Function

Paper: Part A-Calc / Series: 2009 / Difficulty: Easy / Question Number: 2

2. The rate at which people enter an auditorium for a rock concert is modeled by the function R given by $R(t) = 1380t^2 - 675t^3$ for $0 \le t \le 2$ hours; $R(t)$ is measured in people per hour. No one is in the auditorium at time $t = 0$, when the doors open. The doors close and the concert begins at time $t = 2$.

    (a) How many people are in the auditorium when the concert begins?

    (b) Find the time when the rate at which people enter the auditorium is a maximum. Justify your answer.

    (c) The total wait time for all the people in the auditorium is found by adding the time each person waits, starting at the time the person enters the auditorium and ending when the concert begins. The function w models the total wait time for all the people who enter the auditorium before time t. The derivative of w is given by $w'(t) = (2 - t)R(t)$. Find $w(2) - w(1)$, the total wait time for those who enter the auditorium after time $t = 1$.

    (d) On average, how long does a person wait in the auditorium for the concert to begin? Consider all people who enter the auditorium after the doors open, and use the model for total wait time from part (c).

Question 12

Qualification: AP Calculus AB

Areas: Applications of Differentiation, Applications of Integration, Differentiation

Subtopics: Kinematics (Displacement, Velocity, and Acceleration), Increasing/Decreasing , Differentiation Technique – Trigonometry, Average Value of a Function

Paper: Part B-Non-Calc / Series: 2010-Form-B / Difficulty: Somewhat Challenging / Question Number: 6

6. Two particles move along the x-axis. For $0 \le t \le 6$, the position of particle P at time t is given by $p(t) = 2\cos\left(\frac{\pi}{4}t\right)$, while the position of particle R at time t is given by $r(t) = t^3 - 6t^2 + 9t + 3$.

(a) For $0 \le t \le 6$, find all times t during which particle R is moving to the right.

(b) For $0 \le t \le 6$, find all times t during which the two particles travel in opposite directions.

(c) Find the acceleration of particle P at time $t = 3$. Is particle P speeding up, slowing down, or doing neither at time $t = 3$? Explain your reasoning.

(d) Write, but do not evaluate, an expression for the average distance between the two particles on the interval $1 \le t \le 3$.

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Question 13

Qualification: AP Calculus AB

Areas: Applications of Integration

Subtopics: Increasing/Decreasing , Average Value of a Function, Kinematics (Displacement, Velocity, and Acceleration)

Paper: Part A-Calc / Series: 2011 / Difficulty: Easy / Question Number: 1

1. For $0 \le t \le 6$, a particle is moving along the x-axis. The particle's position, $x(t)$, is not explicitly given. The velocity of the particle is given by $v(t) = 2\sin\left(e^{t/4}\right) + 1$. The acceleration of the particle is given by $a(t) = \frac{1}{2}e^{t/4}\cos\left(e^{t/4}\right)$ and $x(0) = 2$.

   (a) Is the speed of the particle increasing or decreasing at time $t = 5.5$? Give a reason for your answer.

   (b) Find the average velocity of the particle for the time period $0 \le t \le 6$.

   (c) Find the total distance traveled by the particle from time $t = 0$ to $t = 6$.

   (d) For $0 \le t \le 6$, the particle changes direction exactly once. Find the position of the particle at that time.

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Question 14

Qualification: AP Calculus AB

Areas: Limits and Continuity, Applications of Integration, Differentiation

Subtopics: Continuities and Discontinuities, Average Value of a Function, Integration Technique – Exponentials, Integration Technique – Trigonometry, Differentiation Technique – Trigonometry, Differentiation Technique – Exponentials

Paper: Part B-Non-Calc / Series: 2011 / Difficulty: Somewhat Challenging / Question Number: 6

6. Let f be a function defined by $f(x) = \begin{cases} 1 - 2\sin x & \text{for } x \le 0 \\ e^{-4x} & \text{for } x > 0. \end{cases}$

(a) Show that f is continuous at $x = 0$.

(b) For $x \ne 0$, express $f'(x)$ as a piecewise-defined function. Find the value of x for which $f'(x) = -3$.

(c) Find the average value of f on the interval $[-1, 1]$.

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Question 15

Qualification: AP Calculus AB

Areas: Limits and Continuity, Applications of Integration

Subtopics: Continuities and Discontinuities, Average Value of a Function, Interpreting Meaning in Applied Contexts, Modelling Situations, Calculating Limits Algebraically, Accumulation of Change

Paper: Part A-Calc / Series: 2011-Form-B / Difficulty: Easy / Question Number: 2

2. A 12,000-liter tank of water is filled to capacity. At time $t = 0$, water begins to drain out of the tank at a rate modeled by $r(t)$, measured in liters per hour, where r is given by the piecewise-defined function

$$r(t) = \begin{cases} \dfrac{600t}{t+3} & \text{for } 0 \le t \le 5 \\ 1000e^{-0.2t} & \text{for } t > 5 \end{cases}$$

(a) Is r continuous at $t = 5$? Show the work that leads to your answer.

(b) Find the average rate at which water is draining from the tank between time $t = 0$ and time $t = 8$ hours.

(c) Find $r'(3)$. Using correct units, explain the meaning of that value in the context of this problem.

(d) Write, but do not solve, an equation involving an integral to find the time A when the amount of water in the tank is 9000 liters.

Question 16

Qualification: AP Calculus AB

Areas: Applications of Integration, Differentiation

Subtopics: Rates of Change (Average), Interpreting Meaning in Applied Contexts, Total Amount, Average Value of a Function, Modelling Situations

Paper: Part A-Calc / Series: 2014 / Difficulty: Medium / Question Number: 1

1. Grass clippings are placed in a bin, where they decompose. For $0 \le t \le 30$, the amount of grass clippings remaining in the bin is modeled by $A(t) = 6.687(0.931)^t$, where $A(t)$ is measured in pounds and t is measured in days.

(a) Find the average rate of change of $A(t)$ over the interval $0 \le t \le 30$. Indicate units of measure.

(b) Find the value of $A'(15)$. Using correct units, interpret the meaning of the value in the context of the problem.

(c) Find the time t for which the amount of grass clippings in the bin is equal to the average amount of grass clippings in the bin over the interval $0 \le t \le 30$.

(d) For $t > 30$, $L(t)$, the linear approximation to A at $t = 30$, is a better model for the amount of grass clippings remaining in the bin. Use $L(t)$ to predict the time at which there will be 0.5 pound of grass clippings remaining in the bin. Show the work that leads to your answer.

Question 17

Qualification: AP Calculus AB

Areas: Applications of Differentiation, Applications of Integration

Subtopics: Rates of Change (Average), Interpreting Meaning in Applied Contexts, Riemann Sums – Right, Kinematics (Displacement, Velocity, and Acceleration), Average Value of a Function

Paper: Part B-Non-Calc / Series: 2015 / Difficulty: Medium / Question Number: 3

t (minutes)	0	12	20	24	40
$v(t)$ (meters per minute)	0	200	240	–220	150

3. Johanna jogs along a straight path. For $0 \le t \le 40$, Johanna's velocity is given by a differentiable function v. Selected values of $v(t)$, where t is measured in minutes and $v(t)$ is measured in meters per minute, are given in the table above.

(a) Use the data in the table to estimate the value of $v'(16)$.

(b) Using correct units, explain the meaning of the definite integral $\int_0^{40} |v(t)|\, dt$ in the context of the problem.

Approximate the value of $\int_0^{40} |v(t)|\, dt$ using a right Riemann sum with the four subintervals indicated in the table.

(c) Bob is riding his bicycle along the same path. For $0 \le t \le 10$, Bob's velocity is modeled by $B(t) = t^3 - 6t^2 + 300$, where t is measured in minutes and $B(t)$ is measured in meters per minute.

Find Bob's acceleration at time $t = 5$.

(d) Based on the model B from part (c), find Bob's average velocity during the interval $0 \le t \le 10$.

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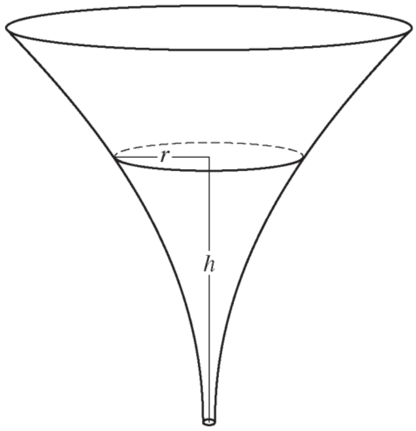
Question 18

Qualification: AP Calculus AB

Areas: Applications of Integration

Subtopics: Average Value of a Function, Volume of Revolution – Disc Method, Rates of Change (Instantaneous), Integration Technique – Standard Functions, Modelling Situations, Related Rates

Paper: Part B-Non-Calc / Series: 2016 / Difficulty: Medium / Question Number: 5



5. The inside of a funnel of height 10 inches has circular cross sections, as shown in the figure above. At height h, the radius of the funnel is given by $r = \frac{1}{20}(3 + h^2)$, where $0 \le h \le 10$. The units of r and h are inches.

(a) Find the average value of the radius of the funnel.

(b) Find the volume of the funnel.

(c) The funnel contains liquid that is draining from the bottom. At the instant when the height of the liquid is $h = 3$ inches, the radius of the surface of the liquid is decreasing at a rate of $\frac{1}{5}$ inch per second. At this instant, what is the rate of change of the height of the liquid with respect to time?

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Question 19

Qualification: AP Calculus AB

Areas: Applications of Differentiation, Applications of Integration

Subtopics: Rates of Change (Average), Mean Value Theorem, Average Value of a Function, Riemann Sums – Trapezoidal Rule, Modelling Situations, Rates of Change (Instantaneous), Related Rates

Paper: Part B-Non-Calc / Series: 2018 / Difficulty: Medium / Question Number: 4

t (years)	2	3	5	7	10
$H(t)$ (meters)	1.5	2	6	11	15

4. The height of a tree at time t is given by a twice-differentiable function H, where $H(t)$ is measured in meters and t is measured in years. Selected values of $H(t)$ are given in the table above.

(a) Use the data in the table to estimate $H'(6)$. Using correct units, interpret the meaning of $H'(6)$ in the context of the problem.

(b) Explain why there must be at least one time t, for $2 < t < 10$, such that $H'(t) = 2$.

(c) Use a trapezoidal sum with the four subintervals indicated by the data in the table to approximate the average height of the tree over the time interval $2 \le t \le 10$.

(d) The height of the tree, in meters, can also be modeled by the function G, given by $G(x) = \dfrac{100x}{1 + x}$, where x is the diameter of the base of the tree, in meters. When the tree is 50 meters tall, the diameter of the base of the tree is increasing at a rate of 0.03 meter per year. According to this model, what is the rate of change of the height of the tree with respect to time, in meters per year, at the time when the tree is 50 meters tall?

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Question 20

Qualification: AP Calculus AB

Areas: Applications of Differentiation, Applications of Integration

Subtopics: Total Amount, Average Value of a Function, Global or Absolute Minima and Maxima, Modelling Situations, Increasing/Decreasing , Accumulation of Change

Paper: Part A-Calc / Series: 2019 / Difficulty: Somewhat Challenging / Question Number: 1

1. Fish enter a lake at a rate modeled by the function E given by $E(t) = 20 + 15\sin\left(\frac{\pi t}{6}\right)$. Fish leave the lake at a rate modeled by the function L given by $L(t) = 4 + 2^{0.1t^2}$. Both $E(t)$ and $L(t)$ are measured in fish per hour, and t is measured in hours since midnight $(t = 0)$.

   (a) How many fish enter the lake over the 5-hour period from midnight $(t = 0)$ to 5 A.M. $(t = 5)$? Give your answer to the nearest whole number.

   (b) What is the average number of fish that leave the lake per hour over the 5-hour period from midnight $(t = 0)$ to 5 A.M. $(t = 5)$?

   (c) At what time t, for $0 \le t \le 8$, is the greatest number of fish in the lake? Justify your answer.

   (d) Is the rate of change in the number of fish in the lake increasing or decreasing at 5 A.M. $(t = 5)$? Explain your reasoning.

Question 21

Qualification: AP Calculus AB

Areas: Applications of Integration, Applications of Differentiation, Integration

Subtopics: Interpreting Meaning in Applied Contexts, Rates of Change (Average), Riemann Sums – Right, Increasing/Decreasing , Differentiation Technique – Chain Rule, Average Value of a Function

Paper: Part A-Calc / Series: 2021 / Difficulty: Somewhat Challenging / Question Number: 1

r (centimeters)	0	1	2	2.5	4
$f(r)$ (milligrams per square centimeter)	1	2	6	10	18

1. The density of a bacteria population in a circular petri dish at a distance r centimeters from the center of the dish is given by an increasing, differentiable function f, where $f(r)$ is measured in milligrams per square centimeter. Values of $f(r)$ for selected values of r are given in the table above.

   (a) Use the data in the table to estimate $f'(2.25)$. Using correct units, interpret the meaning of your answer in the context of this problem.

   (b) The total mass, in milligrams, of bacteria in the petri dish is given by the integral expression $2\pi \int_0^4 r f(r)\, dr$. Approximate the value of $2\pi \int_0^4 r f(r)\, dr$ using a right Riemann sum with the four subintervals indicated by the data in the table.

   (c) Is the approximation found in part (b) an overestimate or underestimate of the total mass of bacteria in the petri dish? Explain your reasoning.

   (d) The density of bacteria in the petri dish, for $1 \le r \le 4$, is modeled by the function g defined by $g(r) = 2 - 16(\cos(1.57\sqrt{r}))^3$. For what value of k, $1 < k < 4$, is $g(k)$ equal to the average value of $g(r)$ on the interval $1 \le r \le 4$?

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Question 22

Qualification: AP Calculus AB

Areas: Applications of Differentiation, Applications of Integration

Subtopics: Total Amount, Average Value of a Function, Increasing/Decreasing , Global or Absolute Minima and Maxima, Fundamental Theorem of Calculus (Second), Accumulation of Change

Paper: Part A-Calc / Series: 2022 / Difficulty: Easy / Question Number: 1

1. From 5 A.M. to 10 A.M., the rate at which vehicles arrive at a certain toll plaza is given by $A(t) = 450\sqrt{\sin(0.62t)}$, where t is the number of hours after 5 A.M. and $A(t)$ is measured in vehicles per hour. Traffic is flowing smoothly at 5 A.M. with no vehicles waiting in line.

   (a) Write, but do not evaluate, an integral expression that gives the total number of vehicles that arrive at the toll plaza from 6 A.M. ($t = 1$) to 10 A.M. ($t = 5$).

   (b) Find the average value of the rate, in vehicles per hour, at which vehicles arrive at the toll plaza from 6 A.M. ($t = 1$) to 10 A.M. ($t = 5$).

   (c) Is the rate at which vehicles arrive at the toll plaza at 6 A.M. ($t = 1$) increasing or decreasing? Give a reason for your answer.

   (d) A line forms whenever $A(t) \geq 400$. The number of vehicles in line at time t, for $a \leq t \leq 4$, is given by $N(t) = \int_a^t (A(x) - 400)\, dx$, where a is the time when a line first begins to form. To the nearest whole number, find the greatest number of vehicles in line at the toll plaza in the time interval $a \leq t \leq 4$. Justify your answer.

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Question 23

Qualification: AP Calculus AB

Areas: Applications of Integration, Integration

Subtopics: Interpreting Meaning in Applied Contexts, Riemann Sums – Right, Mean Value Theorem, Average Value of a Function

Paper: Part A-Calc / Series: 2023 / Difficulty: Easy / Question Number: 1

t (seconds)	0	60	90	120	135	150
$f(t)$ (gallons per second)	0	0.1	0.15	0.1	0.05	0

1. A customer at a gas station is pumping gasoline into a gas tank. The rate of flow of gasoline is modeled by a differentiable function f, where $f(t)$ is measured in gallons per second and t is measured in seconds since pumping began. Selected values of $f(t)$ are given in the table.

(a) Using correct units, interpret the meaning of $\int_{60}^{135} f(t)\,dt$ in the context of the problem. Use a right Riemann sum with the three subintervals [60, 90], [90, 120], and [120, 135] to approximate the value of $\int_{60}^{135} f(t)\,dt$.

(b) Must there exist a value of c, for $60 < c < 120$, such that $f'(c) = 0$? Justify your answer.

(c) The rate of flow of gasoline, in gallons per second, can also be modeled by $g(t) = \left(\frac{t}{500}\right)\cos\left(\left(\frac{t}{120}\right)^2\right)$ for $0 \le t \le 150$. Using this model, find the average rate of flow of gasoline over the time interval $0 \le t \le 150$.

Show the setup for your calculations.

(d) Using the model g defined in part (c), find the value of $g'(140)$. Interpret the meaning of your answer in the context of the problem.